Prove that $\sum_{i=1}^{n} (2i-3)3^{i-1} = (n-2)3^n + 2$ for all positive integers n <u>using mathematical induction</u>.

SCORE: /15 PTS

Basis case:

$$\sum_{i=1}^{n} (2i-3)3^{i-1} = (-1)3^{0} = -1 = (-1)3^{1} + 2$$

GRADED BY

Inductive step:

Assume that
$$\sum_{i=1}^{k} (2i-3)3^{i-1} = (k-2)3^k + 2$$
 for some arbitrary integer $k \ge 1$

ME

Prove that
$$\sum_{i=1}^{k+1} (2i-3)3^{i-1} = (k+1-2)3^{k+1} + 2 = (k-1)3^{k+1} + 2$$

$$= \sum_{i=1}^{k} (2i-3)3^{i-1} + (2(k+1)-3)3^{k}$$

$$= \sum_{i=1}^{k} (2i-3)3^{i-1} + (2k-1)3^{k}$$

$$= (k-2)3^{k} + 2 + (2k-1)3^{k}$$

$$= (k-2)3^{k} + (2k-1)3^{k} + 2$$

$$= (k-2+2k-1)3^{k} + 2$$

$$= (3k-3)3^{k} + 2$$

$$= 3(k-1)3^{k} + 2$$

 $\sum_{i=1}^{k+1} (2i-3)3^{i-1}$

 $=(k-1)3^{k+1}+2$

So, by mathematical induction, $\sum_{i=1}^{n} (2i-3)3^{i-1} = (n-2)3^n + 2$ for all positive integers n

Expand and simplify $(\sqrt{t}-2t^3)^4$. You may write the coefficients in your final answer in factored form, as shown in lecture.

The coefficients in your final answer must **NOT** contain division, ! nor C(n, r) (or equivalent) notation.

NOTE: Do NOT use your calculator's ! nor
$$C(n, r)$$
 features.

$$(\sqrt{t})^4 + 4(\sqrt{t})^3(-2t^3) + 6(\sqrt{t})^2(-2t^3)^2 + 4(\sqrt{t})(-2t^3)^3 + (-2t^3)^4$$

SCORE:

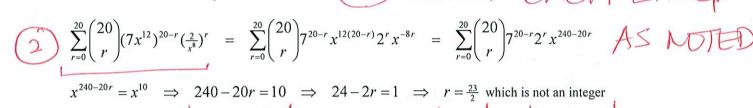
Consider the expansion of $(7x^{12} + \frac{2}{x^8})^{20}$. SCORE: _____/9 PTS

For the questions below, you may write the coefficients in your final answers in factored form, as shown in lecture.

The coefficients in your final answers must **NOT** contain division, ! nor C(n, r) (or equivalent) notation.

NOTE: Do NOT use your calculator's ! nor C(n, r) features.

[a] Find the coefficient of
$$x^{10}$$
 in the expansion.



No
$$x^{10}$$
 term, so coefficient = 0

[b] Find the coefficient of x^{-80} in the expansion.

$$x^{240-20r} = x^{-80} \implies 240-20r = -80 \implies 24-2r = -8 \implies r = 16$$

$$\binom{20}{16}7^{20-16}2^{16} = \binom{20}{16}7^42^{16} = \boxed{\frac{20!}{16!4!}}7^42^{16} = \boxed{\frac{20\cdot19\cdot18\cdot17\cdot16!}{16!\cdot4\cdot3\cdot2\cdot1}}7^42^{16} = 5\cdot19\cdot3\cdot17\cdot7^42^{16}$$

[c] Find the sixth term in the expansion.